SIGGRAPH ASIA 2025 香港 HONG KONG

Frame-Free Representation of Polarized Light for Resolving Stokes vector Singularities

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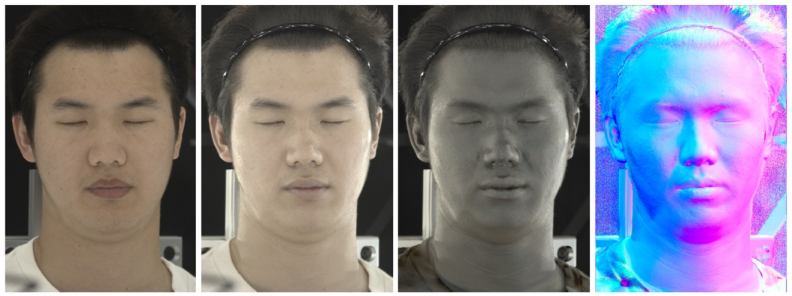
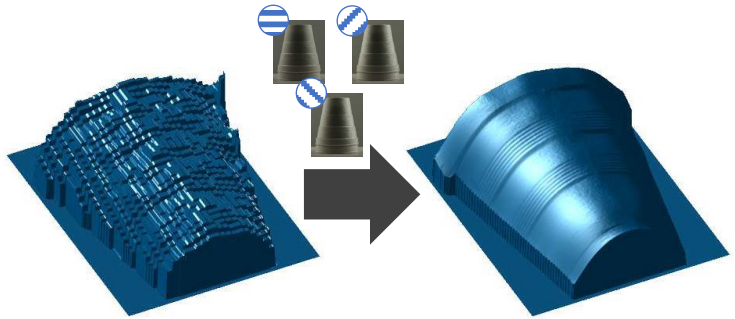
Sponsored by acm

Organized by koelnmesse

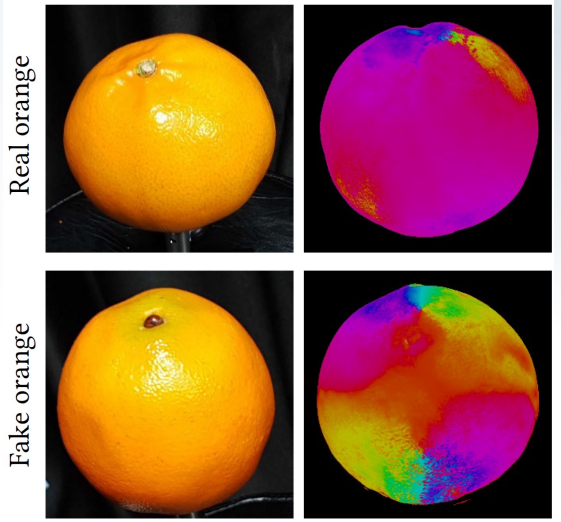
Polarization: Motivation



[Ghosh et al. 2011]



[Kadambi et al. 2015] etc.



[Hwang et al. 2022]

Separate diffuse and specular

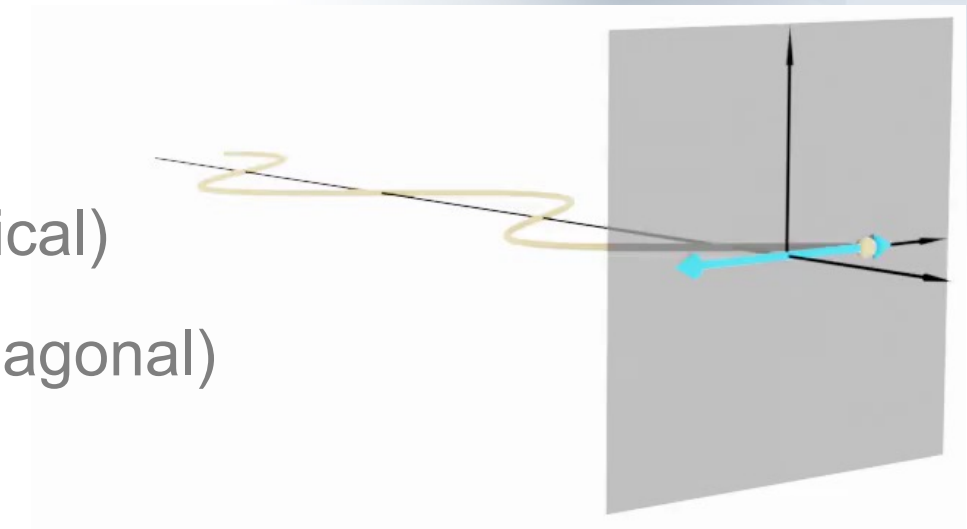
Fine-scale surface normal

Distinguish materials

Polarization in Stokes parameters

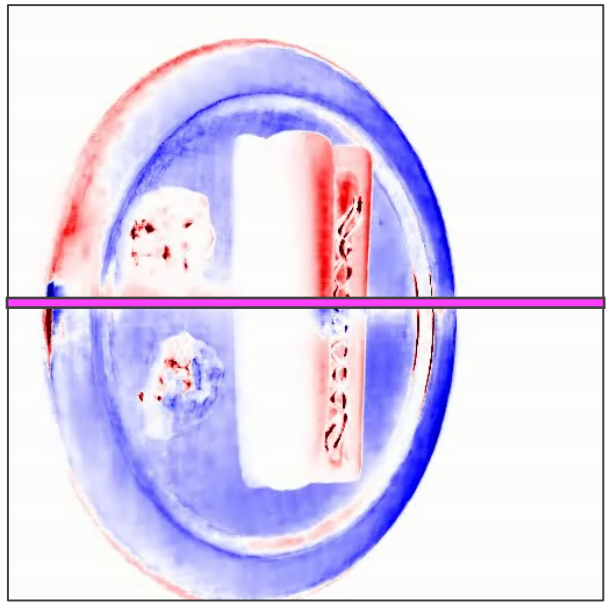
$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

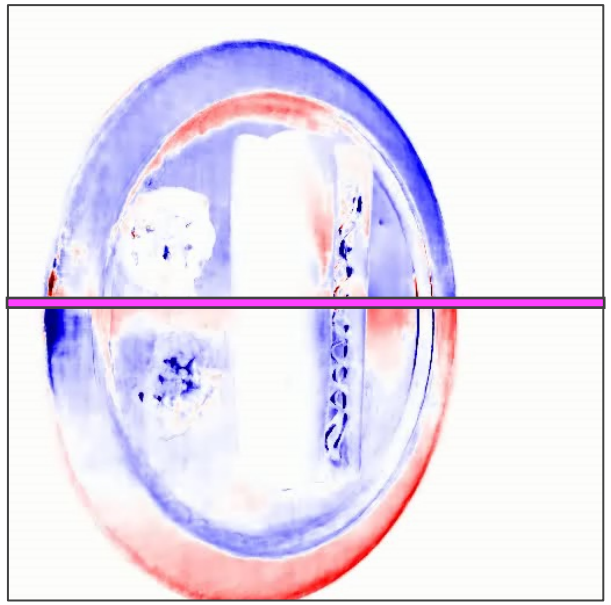
- S_0 total intensity
- S_1 linear polarization (horizontal / vertical)
- S_2 linear polarization (diagonal / antidiagonal)
- S_3 circular polarization

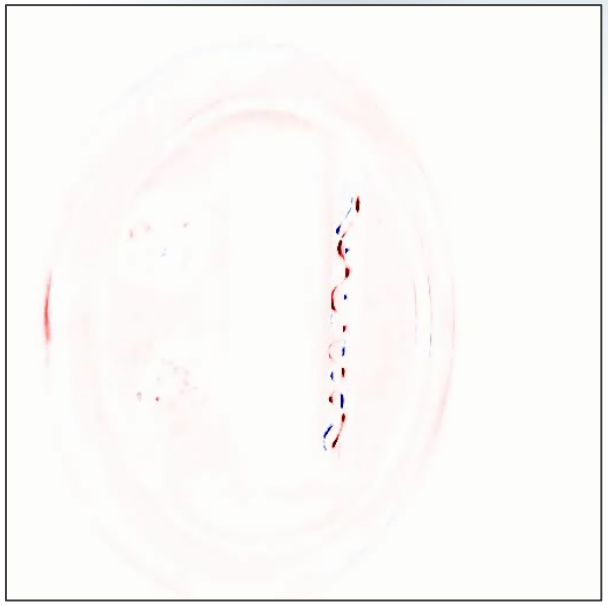


Challenge: Singularity in Polarized Radiance Fields

NeSpoF [Kim et al. 2023]







s_0 s_1 s_2 s_3

Challenge: Singularity in Polarized Radiance Fields

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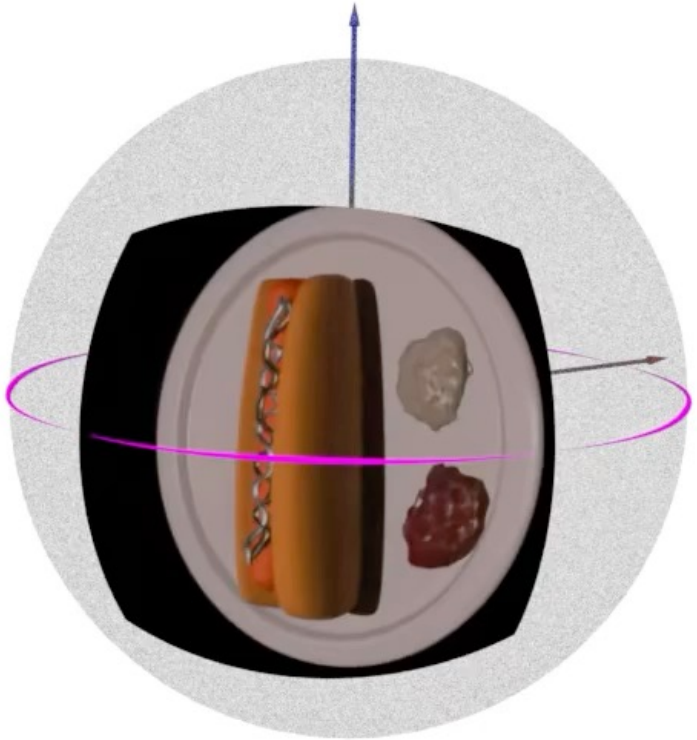
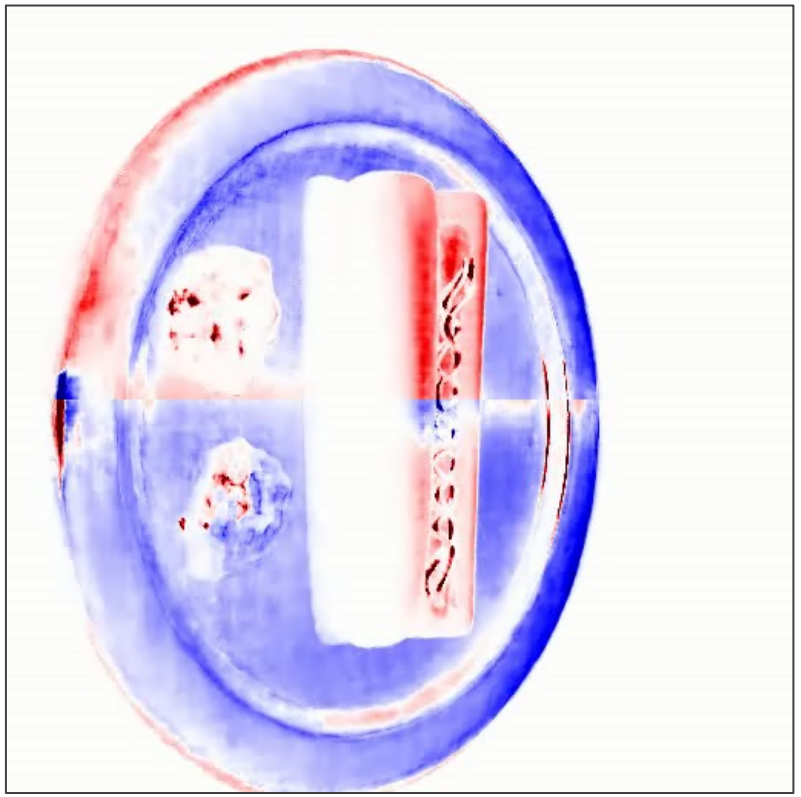
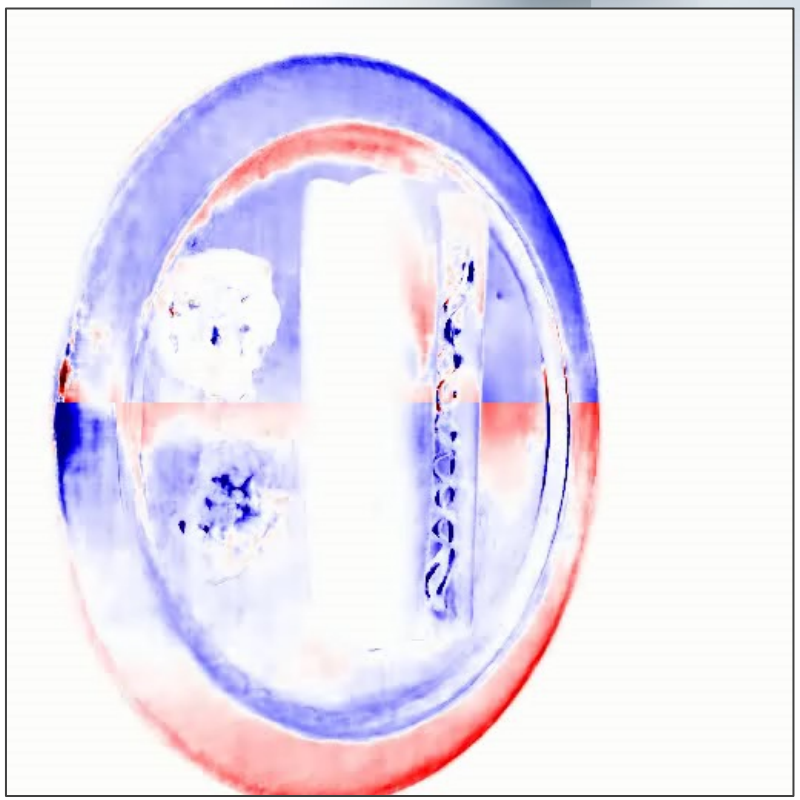




S_1



S_2

* This artifact was not visible in [Kim et al. 2023]
* But become visible after world coordinate rotation

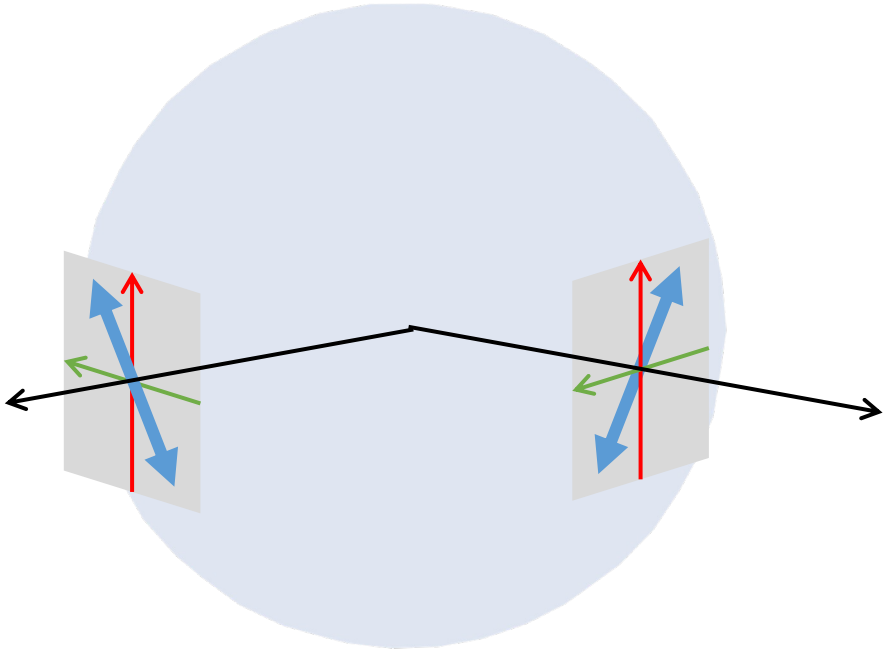
Challenge: Singularity in Stokes Parameters

Why Do Such Singularities Occur?

Not a mistake, bug, technical details…

Fundamental limitation to represent polarized light

Frame for Each Ray



e.g.

- Radiance fields (fixed position)
- Environment light

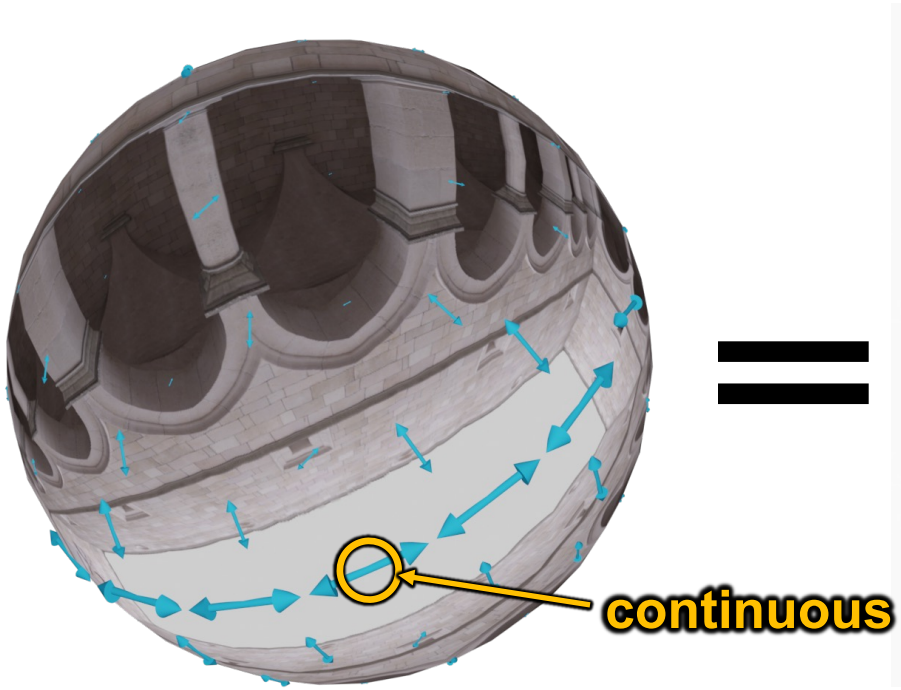
Stokes vectors on the directional domain

Challenge: Singularity in Stokes Parameters

Stokes parameters w.r.t frame

Stokes vector field $\vec{f}$

Frame field $\vec{\mathbf{F}}(\hat{\omega})$



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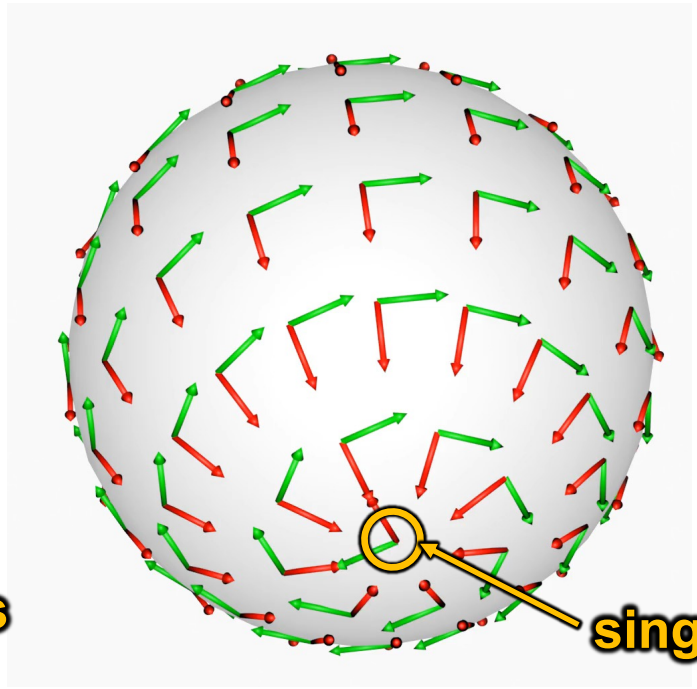

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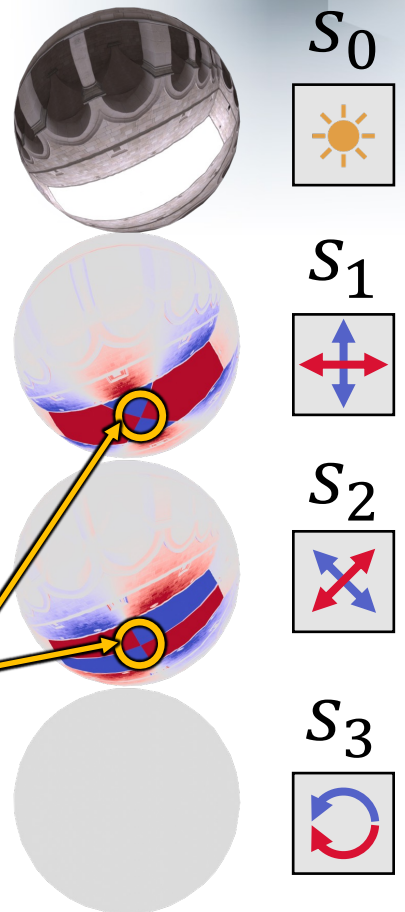
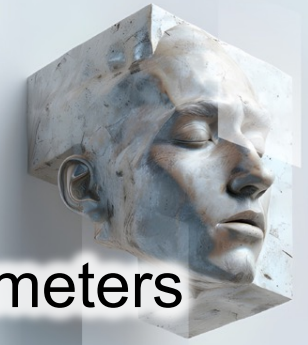


Problem: singularity

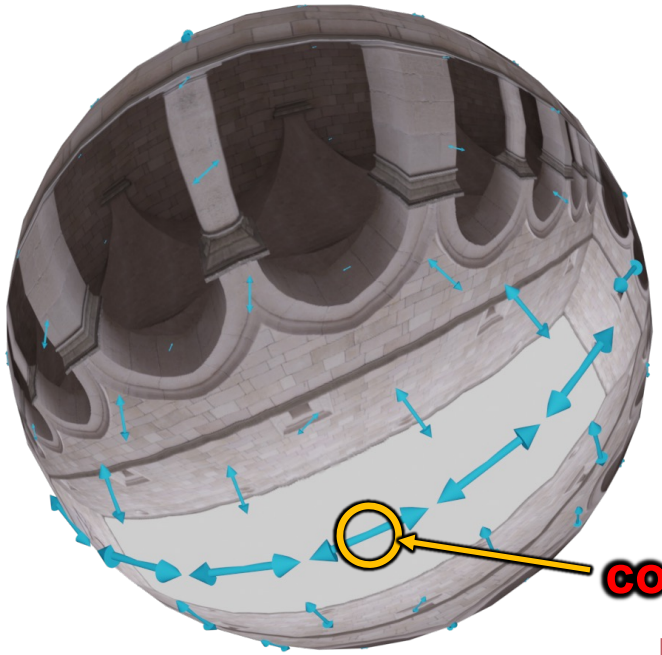
Not avoidable due to the Hairy ball theorem

Our Goal: Find a Frame-Free Representation



New parameters

Stokes vector field $\overleftrightarrow{f}$



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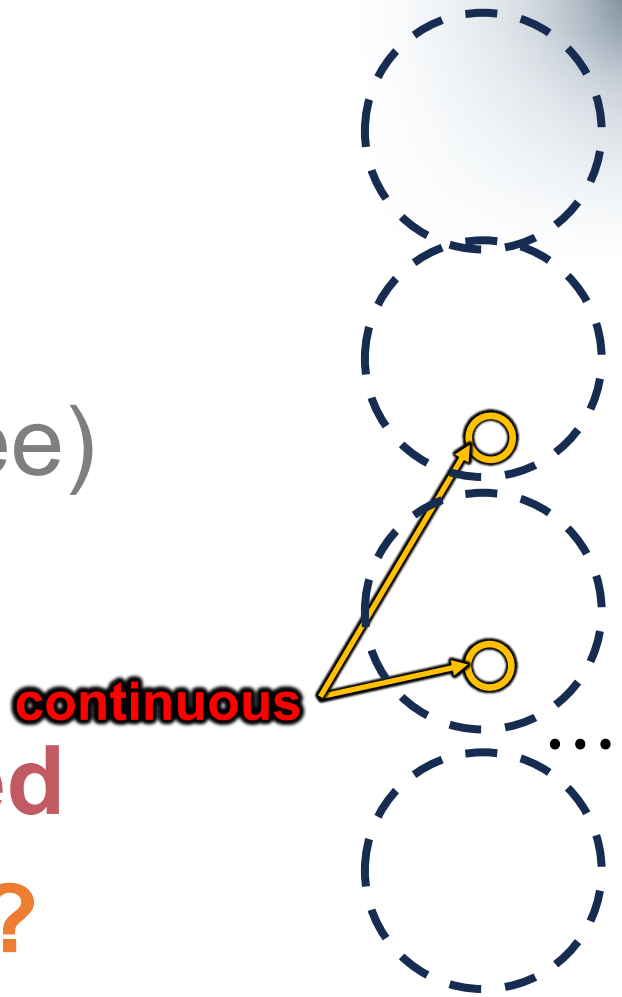
(Frame-Free)

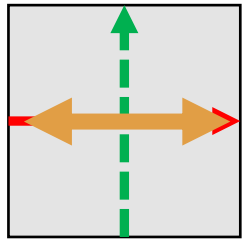


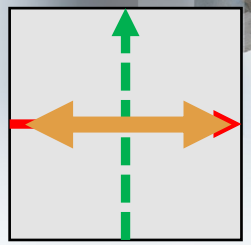
???

Never been existed

Why challenging?

Twice Rotation Property





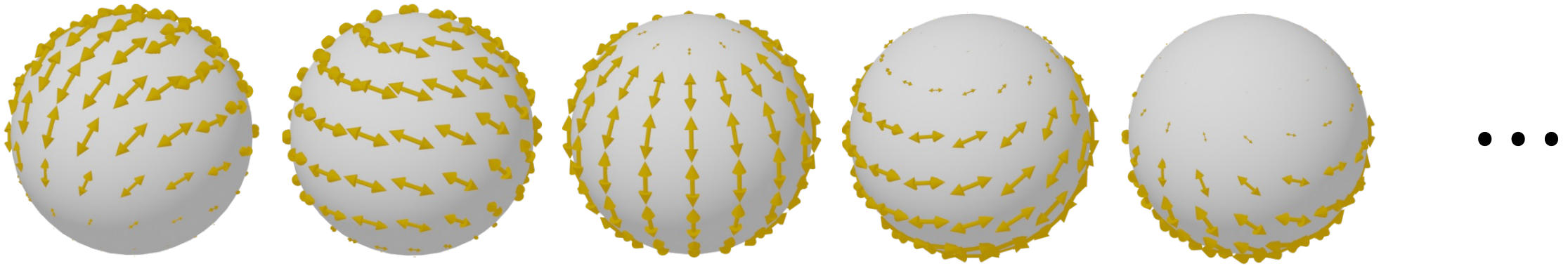
$$\begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} : \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\vartheta & \sin 2\vartheta & 0 \\ 0 & -\sin 2\vartheta & \cos 2\vartheta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ \cos 2\vartheta \\ -\sin 2\vartheta \\ 0 \end{bmatrix}$$

Essentially different from both scalars and vectors

Our Approach

Spin-2 Spherical Harmonics

- Developed in physics [Newman and Penrose 1966]
- Introduced to computer graphics field
  by [Yi et al. 2024] (SIG'24)



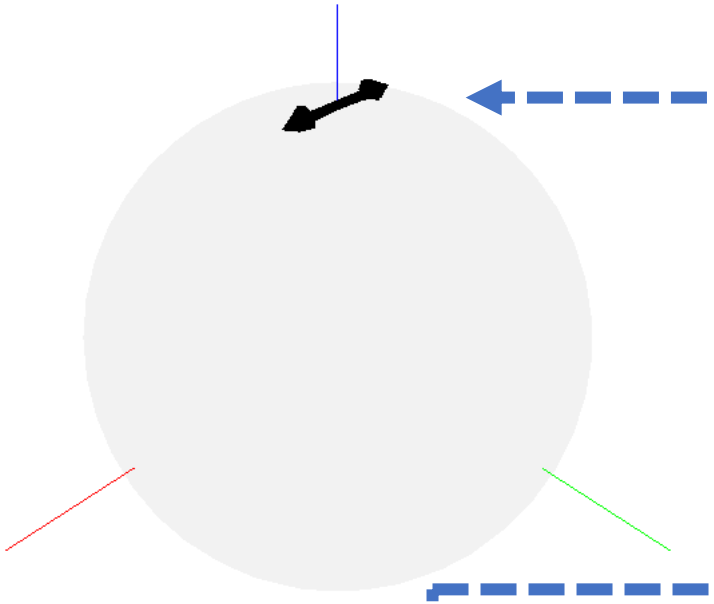
Our Approach

Spin-2 Spherical Harmonics Expansion

$$\begin{pmatrix} f_{2,-21} \\ +f_{2,-22}i \end{pmatrix} + \begin{pmatrix} f_{2,-11} \\ +f_{2,-12}i \end{pmatrix} + \cdots$$

$$= \begin{pmatrix} f_{201} \\ +f_{202}i \end{pmatrix} + \begin{pmatrix} f_{211} \\ +f_{212}i \end{pmatrix} + \begin{pmatrix} f_{221} \\ +f_{222}i \end{pmatrix} + \cdots$$

Cut up to order $l = 2$

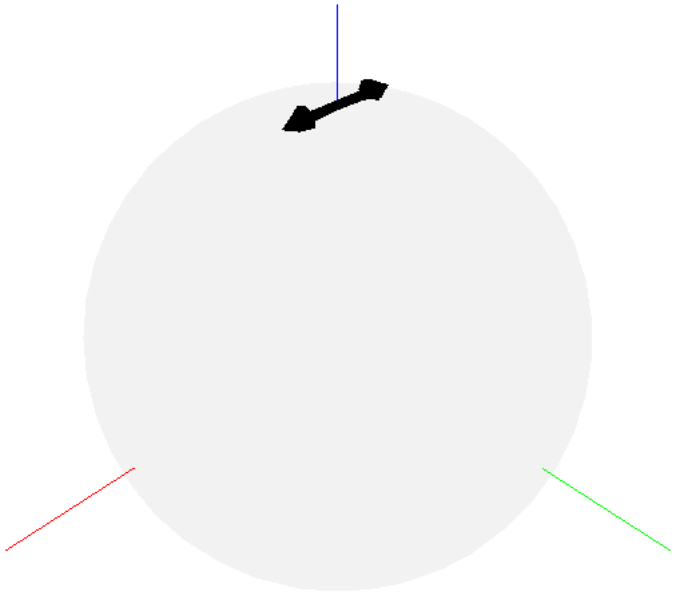


Single Stokes vector = Dirac delta Stokes vector field

Our **S2L2 representation**

- ✓ Frame-free ”global” representaion
- ✓ Invertible

Our Approach



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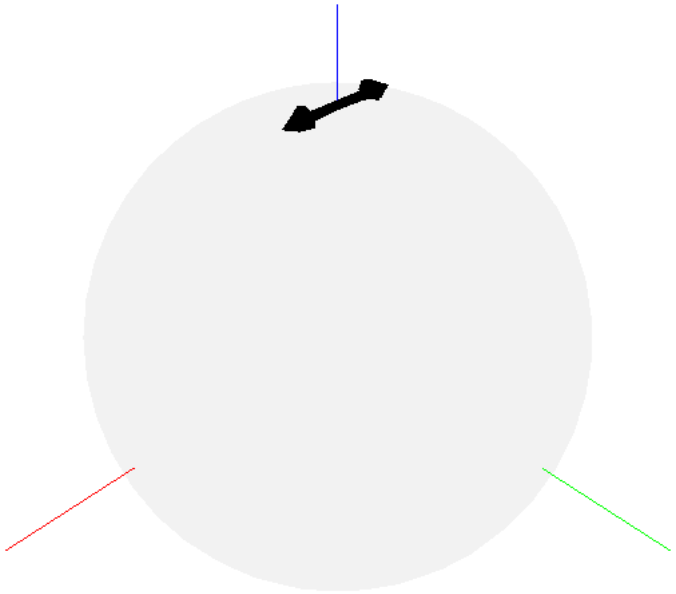

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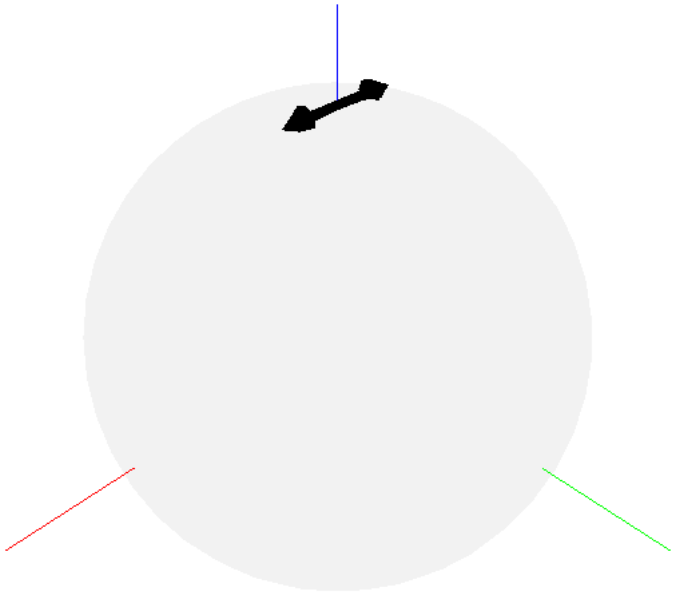
- ✓ Linear (addition preserved)
- ✓ Provide a rotation invariant metric

Our Approach



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All proofs in the paper!

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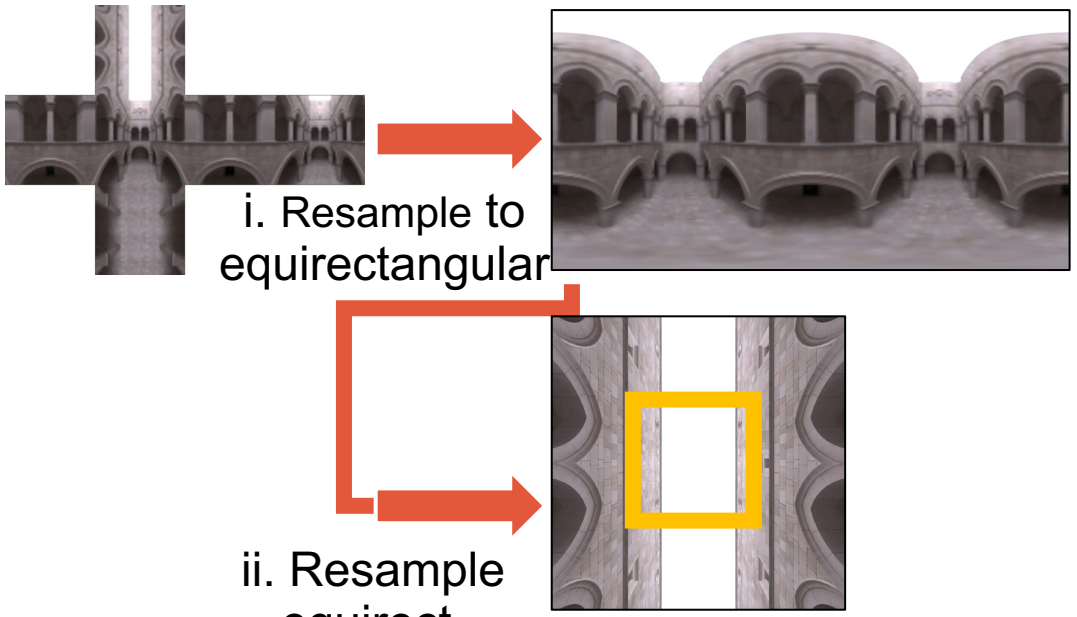
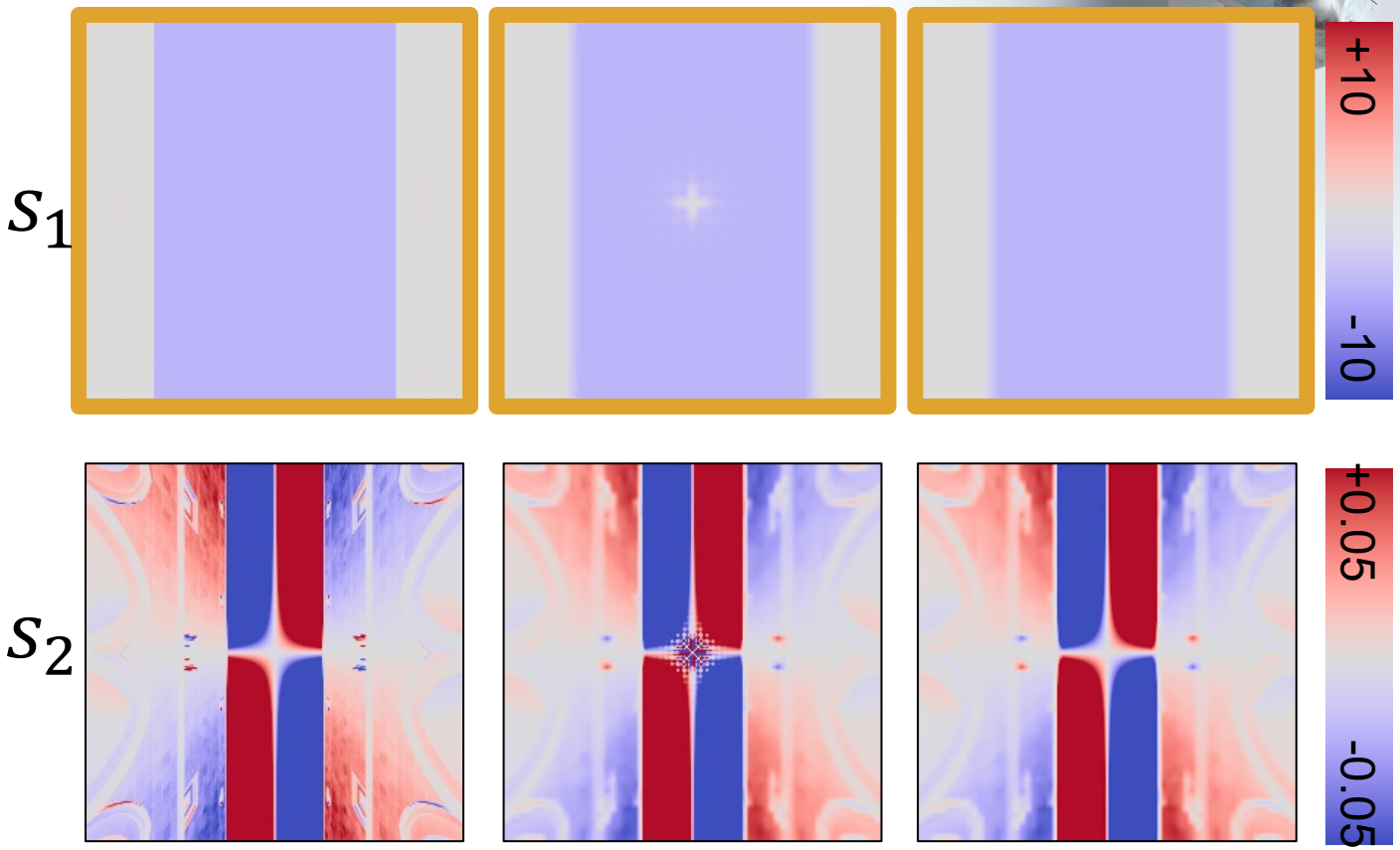
Results

Result: S2L2 Parameters for Stokes Vectors

Application 1: Polarized Radiance Fields

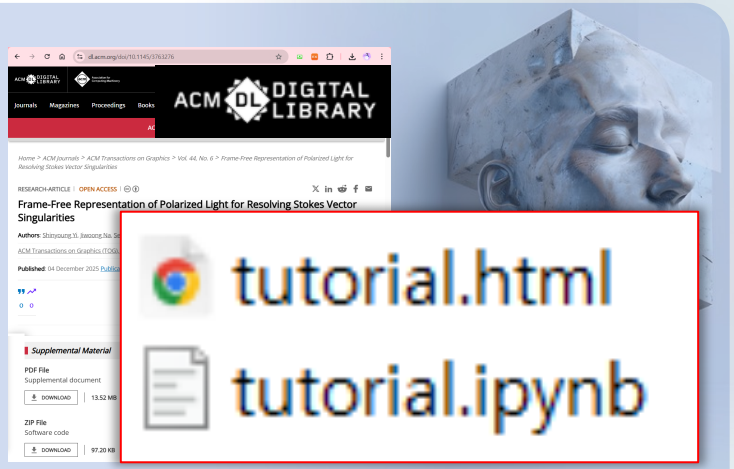
Application 2: Polarized Environment Map Resampling





Conclusion

- Frame-free parameters for Stokes vectors
  - Proofs for guaranteed properties
- Beneficial when continuity over directions is important
- Future work
  - Reducing dimensionality 12 → ?



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Thank you